Comment on "Constraint Quantization of Open String in Background B field and Noncommutative D-brane"

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Abstract

In the paper "Constraint Quantization of Open String in Background B field and Noncommutative D-brane", it is claimed that the boundary conditions lead to an infinite set of secondary constraints and Dirac brackets result in a non-commutative Poisson structure for D-brain. Here we show that contrary to the arguments in that paper, the set of secondary constraints on the boundary is finite and the non-commutativity algebra can not be obtained by evaluating the Dirac brackets.

In ref. [1], Chong-Sun Chu and Pei-Ming Ho have studied the constraint quantization of open string in background B field. They have obtained an infinite set of secondary constraints due to the boundary conditions for the open string on D-brane. Then, they have shown that the Dirac brackets result in the non-commutativity algebra derived in ref. [2] for the end points of the open string and consequently the D-brane becomes non-commutative. This result is very interesting because, as far as we know, this is one of the most important applications of the Dirac method of quantization of the secondary constraints [3]. Here, we show that the Dirac method does not lead to an infinite set of secondary constraints but to a set of finite second class constraint chains which does not lead to the non-commutativity algebra.

The action for an open string ending on a Dp-brane is [1, 2]

$$S_B = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\left[g^{\alpha\beta}G_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu + \mathcal{F}_{ij}\partial_\alpha X^i\partial_\beta X^j\right], \tag{1}$$

where

$$\mathcal{F} = B - dA = B - F, \tag{2}$$

is the modified Born-Infeld field strength. The equation of motion is

$$(\partial_\tau^2 - \partial_\sigma^2)X^\mu = 0, \tag{3}$$

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and the boundary conditions at $\sigma = 0, \pi$ are:

$$\partial_\sigma X^i + \partial_\tau X^j \mathcal{F}_j{}^i = 0, \quad i, j = 0, 1, \cdots, p, \tag{4}$$

$$X^a = x_0^a, \quad a = p+1, \cdots, D. \tag{5}$$

In the following, for simplicity and without loss of generality, we assume the case $p = D$. Defining the momentum fields

$$\Pi_i(\tau, \sigma) = \frac{\delta}{\delta X^i(\sigma, \tau)} S_B = \frac{1}{2\pi\alpha'} \left(\partial_\tau X_i + \partial_\sigma X^j \mathcal{F}_{ji} \right), \tag{6}$$

then the Hamiltonian is

$$H = \frac{1}{4\pi\alpha'} \int d\sigma \left[(2\pi\alpha'\Pi - \partial_\sigma X.\mathcal{F})^2 + (\partial_\sigma X)^2 \right] + \lambda_i^0 \Phi_0^i + \lambda_i^\pi \Phi_\pi^i, \tag{7}$$

where $M_{ij} = \eta_{ij} - \mathcal{F}_i{}^k \mathcal{F}_{kj}$, λ_i^σ's are Lagrange multipliers and Φ_σ^i's are the primary constraints corresponding to the boundary conditions given in Eq.(4) [1, 4]:

$$\Phi_\sigma^i = \int d\sigma' \delta(\sigma' - \sigma) \phi(\sigma'), \qquad \sigma = 0, \pi, \tag{8}$$

in which

$$\phi(\sigma) = 2\pi\alpha' \Pi^k(\sigma) \mathcal{F}_k{}^i + \partial_\sigma X^j(\sigma) M_j{}^i. \tag{9}$$

The secondary constraints can be obtained by considering the consistency conditions of the primary constraints:

$$\dot{\Phi}_\sigma^i = 0 \rightarrow \Psi_\sigma^i = \int d\sigma' \delta(\sigma - \sigma') \psi(\sigma') = 0, \qquad \sigma = 0, \pi, \tag{10}$$

where

$$\psi(\sigma) = \partial_\sigma \Pi(\sigma). \tag{11}$$

This result is the direct consequence of the fact that (see Eq.(28) in ref.[1]),

$$\begin{aligned}
\frac{1}{2\pi\alpha'} \{\phi^i(\tau, \sigma), \phi^j(\tau, \sigma')\} &= -\partial_{\sigma'} \delta(\sigma - \sigma') \mathcal{F}_k{}^i M_{k'}{}^j \eta^{kk'} + \partial_\sigma \delta(\sigma - \sigma') M_k{}^i \mathcal{F}_{k'}{}^j \eta^{kk'} \\
&= \partial_\sigma \delta(\sigma - \sigma') \left(\mathcal{F}^{ki} M_k{}^j + M^{ki} \mathcal{F}_k{}^j \right) \\
&= \partial_\sigma \delta(\sigma - \sigma') \left(-\mathcal{F}M + M\mathcal{F} \right)^{ij} \\
&= 0,
\end{aligned} \tag{12}$$

and consequently,

$$\{\Phi_\sigma^i, \Phi_{\sigma'}^j\} = 0, \qquad \sigma, \sigma' = 0, \pi. \tag{13}$$

[1]To obtain the final result given in Eq.(12) we have used the following properties:

$$\begin{aligned}
\{X^i(\tau, \sigma), \Pi^j(\tau, \sigma')\} &= \delta(\sigma - \sigma') \eta^{ij}, \\
(\mathcal{F})^{ij} &= -(\mathcal{F})^{ji}, \\
(M)^{ij} &= +(M)^{ji}.
\end{aligned} \tag{14}$$

[1]It is worth noting that if we had $\det\left(\{\Phi_\sigma^i, \Phi_{\sigma'}^j\}\right) \neq 0$, then no secondary constraint should be introduced, since the consistency conditions $\dot{\Phi}_\sigma^i = 0$ would determine the Lagrange multipliers [3].

Since

$$\{\phi^i(\tau,\sigma),\psi^j(\tau,\sigma')\} = M^{ij}\partial_\sigma\partial_{\sigma'}\delta(\sigma-\sigma') \neq 0, \tag{15}$$

the constraints Φ^i_σ's and Ψ^i_σ's form a set of secondary constraints [1, 4]. Consequently the consistency of the secondary constraints Ψ^i_σ's determine the Lagrange multiplier and according to the well known arguments in the context of constrained systems, no additional constraint emerges. To calculate the Dirac brackets, it is suitable to define constraints

$$\Omega^a_\sigma = \int d\sigma'\delta(\sigma-\sigma')\omega^a(\sigma'), \qquad a = 1,\cdots,2D, \tag{16}$$

for $\sigma = 0,\pi$, where ω^a's are defined as follows:

$$\begin{aligned} \omega^i &= \phi^i, \\ \omega^{D+i} &= \psi^i, \qquad i = 1,\cdots,D. \end{aligned} \tag{17}$$

The matrix of the Poisson brackets of the constraints Ω^a_0's is

$$C = \begin{pmatrix} 0 & M \\ -M & 0 \end{pmatrix} \int d\sigma d\sigma'\delta(\sigma)\delta(\sigma')\partial_\sigma\partial_{\sigma'}\delta(\sigma-\sigma'). \tag{18}$$

Using the equality

$$\delta(\sigma-\sigma') = \lim_{\epsilon\to 0} \frac{1}{\epsilon\sqrt{\pi}} e^{\frac{-(\sigma-\sigma')^2}{\epsilon^2}}, \tag{19}$$

the inverse of the matrix C can be obtained as follows:

$$C^{-1} = \lim_{\epsilon\to 0} \frac{\epsilon^3\sqrt{\pi}}{2} \begin{pmatrix} 0 & -M^{-1} \\ M^{-1} & 0 \end{pmatrix}. \tag{20}$$

By definition, the Dirac bracket of the fields $X^i(\tau,0)$ is:

$$\begin{aligned} \{X^i(\tau,0),X^j(\tau,0)\}_{DB} &= -\int d\sigma d\sigma'\{X^i(\tau,0),\omega^a(\tau,\sigma)\}C^{-1}_{ab}\{\omega^b(\tau,\sigma'),X^j(\tau,0)\} \\ &\sim \int d\sigma d\sigma'\delta(\sigma)\partial_{\sigma'}\delta(\sigma') = 0. \end{aligned} \tag{21}$$

Consequently the Dirac method of constraint quantization leads to a commutative Poisson structure for D-branes to which the open string end points are attached [5]. Finally it is necessary to note that the final result given in Eq.(21) does not change if one insists on the assertion that an infinite set of constraints exist on the boundaries [4].

References

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